ABSTRACT: MIDLANDS LOGIC SEMINAR, BIRMINGHAM, NOVEMBER 8, 2013.

TRISTRAM DE PIRO

I will give a proof of the Fourier Inversion Theorem for functions $f : \mathcal{R} \to \mathcal{R}$, belonging to the Schwartz class, $(*)$, using the methods of nonstandard analysis, see [3]. The assumption $(*)$ is stronger than the actual hypotheses on f required in the proof, and it is an interesting question as to whether it can be relaxed to only $f, \hat{f} \in L^1 \cap C$; for which a standard proof is known, see [2].

The Fourier transform method is a classical tool in solving the heat equation PDE on $\mathcal{R} \times \mathcal{R}_{\geq 0}$, for a boundary condition f in the Schwartz class (or $L^1 \cap C$), see [5]. However, it is known that the solution obtained, involving the heat kernel Φ, works even for continuous boundary condition, satisfying the growth condition $|f(x)| \leq Aexp(B|x|^{\rho})$, $\rho < 2$, $(**)$, see [1]. I will show how the nonstandard Fourier transform can be used to provide a solution, with just this assumption $(**)$, see [4].

References

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Mathematics Department, Harrison Building, Streatham Campus, University of Exeter, North Park Road, Exeter, Devon, EX4 4QF, United Kingdom

E-mail address: tdpd201@exeter.ac.uk